

## >> Chapter 11 Appendix: Deriving the Multiplier Algebraically

This appendix shows how to derive the multiplier algebraically. First recall that in this chapter planned aggregate spending,  $AE_{Planned}$  is the sum of consumer spending,  $C$ , which is determined by the consumption function, and planned investment spending,  $I_{Planned}$ . Rewriting equation 11-9 to express all its terms fully, we have:

$$(11A-1) \quad AE_{Planned} = A + MPC \times YD + I_{Planned}$$

Because there are no taxes or government transfers in this model, disposable income is equal to GDP, so equation 11A-1 becomes

$$(11A-2) \quad AE_{Planned} = A + MPC \times GDP + I_{Planned}$$

The income-expenditure equilibrium GDP,  $Y^*$ , is equal to planned aggregate spending:

$$(11A-3) \quad \begin{aligned} Y^* &= AE_{Planned} \\ &= A + MPC \times Y^* + I_{Planned} \\ &\text{in income-expenditure equilibrium} \end{aligned}$$

Just two more steps. Subtract  $MPC \times Y^*$  from both sides of Equation 11-12:

$$(11A-4) \quad Y^* - MPC \times Y^* = Y^* \times (1 - MPC) = A + I_{Planned}$$

Finally, divide both sides by  $(1 - MPC)$ :

$$(11A-5) \quad Y^* = \frac{A + I_{Planned}}{1 - MPC}$$

Equation 11A-5 tells us that a \$1 autonomous change in planned aggregate spending—a change in either  $A$  or  $I_{Planned}$ —causes a  $\$1/(1 - MPC)$  change in income-expenditure equilibrium GDP,  $Y^*$ . The multiplier in our simple model is therefore:

$$(11A-6) \quad \text{Multiplier} = 1/(1 - MPC)$$

### PROBLEMS

- In an economy without government purchases, government transfers, or taxes, aggregate autonomous consumer spending is \$500 billion, planned investment spending is \$250 billion, and the marginal propensity to consume is 0.5.
  - Write the expression for planned aggregate spending as in Equation 11A-1.
  - Solve for  $Y^*$  algebraically.
  - What is the value of the multiplier?
  - How will  $Y^*$  change if autonomous consumer spending falls to \$450 billion?
- Compare the following table by calculating the value of the multiplier and identifying the change in  $Y^*$  due to the change in autonomous spending. How does the value of the multiplier change with the marginal propensity to consume?

MPC	Value of multiplier	Change in spending	Change in $Y^*$
0.5		$\Delta C = + \$50$ million	
0.6		$\Delta I = - \$10$ million	
0.75		$\Delta C = - \$25$ million	
0.8		$\Delta I = + \$20$ million	
0.9		$\Delta C = - \$2.5$ million	